

The Electric Field:

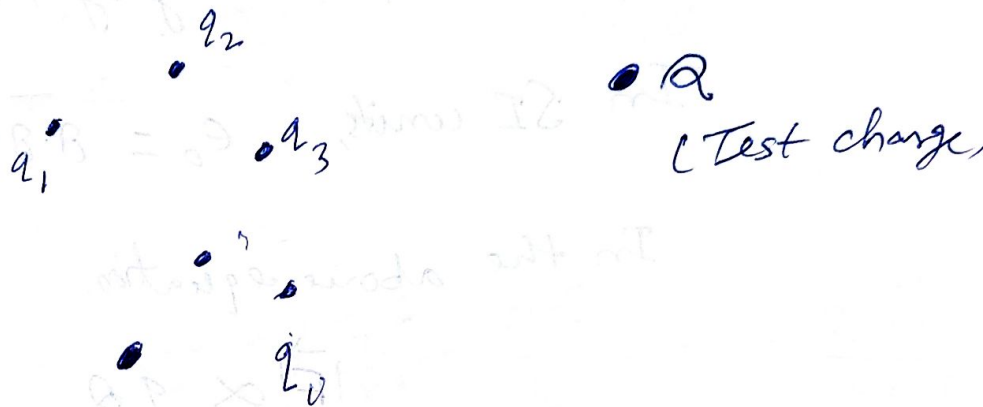
Electrostatics — Study of electromagnetic phenomena when charges are stationary

$q_1, q_2, q_3, \dots, q_i$ are electric charges.

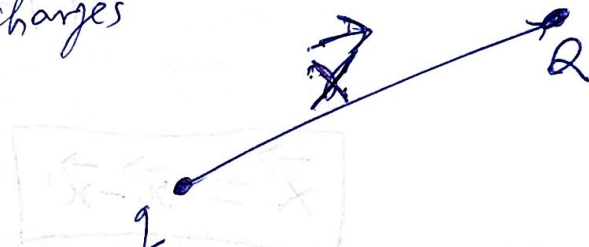
→ We call them Source charges.

Q → Test charge

Source charges



→ What will be the force on test charge Q due to the source charges?



This can be calculated using 'principle of superposition' → which says that force between two charges is completely unaffected by presence of other charges.

Now Force on Q due to source charges q_1, q_2, \dots, q_n

will be, $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n$

$\vec{F}_v \rightarrow$ Force on Q due to source charge q_i

Coulomb's law! Force on a test charge Q due to a point charge q (at rest) at a distance r away

is given by

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \hat{x}$$

This is Coulomb's law and is based on experiments.

$\epsilon_0 \rightarrow$ permittivity of free space

In SI units, $\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

In the above equation

$$|\vec{F}| \propto qQ$$

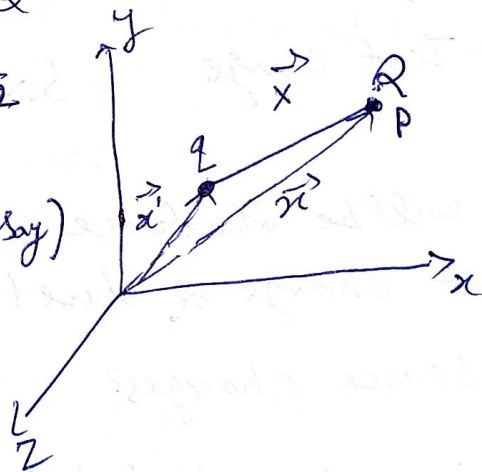
$$|\vec{F}| \propto \frac{1}{r^2}$$

$$\vec{x} = \vec{x}_2 - \vec{x}_1 = r \text{ (say)}$$

$\vec{x} \rightarrow$ Separation vector from \vec{x}_1 to \vec{x}_2

$$\text{and } x = |\vec{x}| = |\vec{x}_2 - \vec{x}_1| = r \text{ (let)}$$

$\hat{x} \rightarrow$ unit vector



Electric Field:

We consider many point charges q_1, q_2, \dots, q_n at distance $r_1, r_2, r_3, \dots, r_n$ from charge Q .

According to the Superposition principle, the total force on Q is given by.

$$\begin{aligned} \vec{F} &= \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n \\ &= \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 Q}{r_1^2} \hat{r}_1 + \frac{q_2 Q}{r_2^2} \hat{r}_2 + \dots + \frac{q_n Q}{r_n^2} \hat{r}_n \right) \\ &= \frac{Q}{4\pi\epsilon_0} \left[\frac{q_1 \hat{r}_1}{r_1^2} + \frac{q_2 \hat{r}_2}{r_2^2} + \dots + \frac{q_n \hat{r}_n}{r_n^2} \right] \end{aligned}$$

or $\vec{F} = Q \vec{E}$

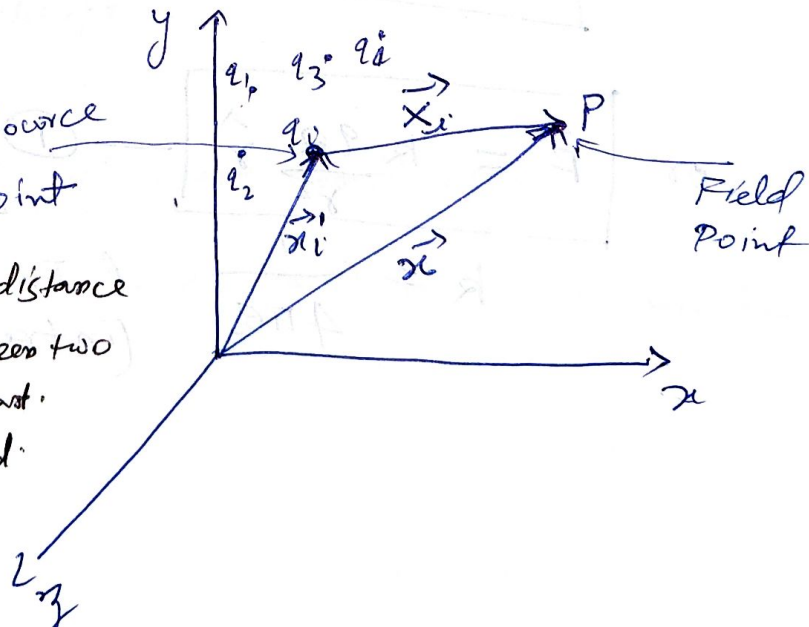
where $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 \hat{r}_1}{r_1^2} + \frac{q_2 \hat{r}_2}{r_2^2} + \dots + \frac{q_n \hat{r}_n}{r_n^2} \right]$

$$\text{or } \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i \hat{r}_i}{r_i^2}$$

\vec{E} is called electric field ~~force~~ of the source charges

$\vec{x}_i = \vec{x} - \vec{x}'_i = \vec{r}_i$ (let) Source

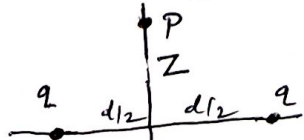
$|\vec{x}_i| = |\vec{x} - \vec{x}'_i| = r_i = r_i$ Point



H.W. Find electric field a distance

z above the midpoint between two equal charges, q , a distance d apart.

check what happens when $z \gg d$ (see P17)



Dr. S.K. Yadav
email: sunil.phy30@gmail.com

Classical Electrostatics: Static charges create static electric fields.

Classical magnetostatics: steadily moving charges create static magnetic field

Classical Electrodynamics: Accelerating charges create (Maxwell Equations) changing fields

Classical means here — macroscopic theory

Coulomb's law:

$$\text{Force (F)} \propto qQ$$

$$F \propto \frac{1}{r^2}$$

$$F \propto \frac{qQ}{r^2}$$

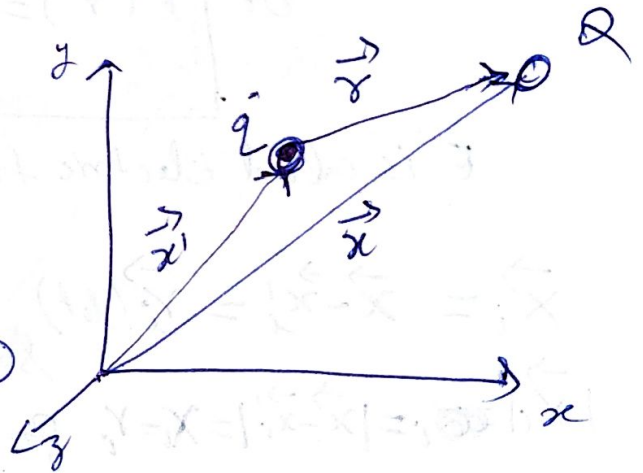
$$\text{or } \vec{F} \propto \frac{qQ}{r^2} \hat{r}$$

$$\vec{r} = \vec{x} - \vec{x}'$$

$$\text{or } \vec{F} = k \frac{qQ}{r^2} \hat{r}$$

$$k = \frac{1}{4\pi\epsilon_0}$$

(In SI unit)
(in free space)



Coulomb's law

(02)

$$\vec{F} = k \frac{qQ}{r^2} \frac{\vec{r}}{r}$$

or $\vec{F} = k \frac{qQ}{r^3} \vec{r}$ — (2)

$$\vec{r} = \vec{x} - \vec{x}'$$

$$r = |\vec{r}| = |\vec{x} - \vec{x}'|$$

Eq. (1) & (2) both are same.

$$\vec{F} = q \cdot \vec{E} = q \cdot k \frac{q \vec{r}}{r^3}$$

$$\vec{E} = k \frac{q \vec{r}}{r^3}$$

or

$$\vec{E} = k q \frac{(\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3}$$

For n point charges $\{q_i\} = q_1, q_2, \dots, q_n$

$$\vec{E} = k \sum_{i=1}^n q_i \frac{(\vec{x} - \vec{x}_i)}{|\vec{x} - \vec{x}_i|^3}$$

Note: ~~Before~~ (i) Since electric field is a vector field, it is defined at every point in space and therefore can be written in terms of spatial coordinates.

(ii) Force is not a field of vectors. This is the force felt at the point of particular charge q_i .

Before moving to notion of ~~charge~~ continuous charge distribution we review here the Dirac Delta function.

One Dimensional Delta function

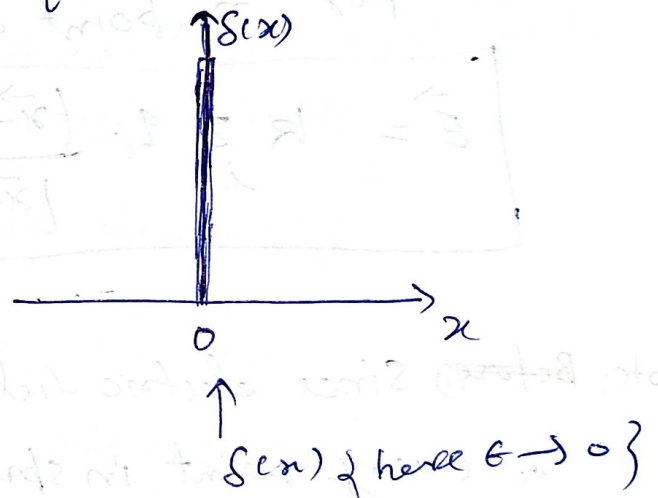
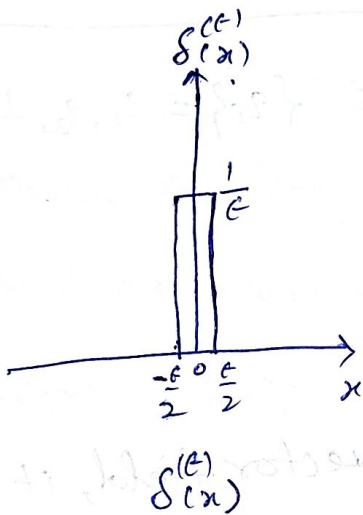
We denote it by $\delta(x)$ in 1D.

There are various definitions of the delta function.

a) Delta function can be defined as the limit of $\delta^{(\epsilon)}(x)$ when $\epsilon \rightarrow 0$ as { see fig. }

$$\delta(x) = \lim_{\epsilon \rightarrow 0} \delta^{(\epsilon)}(x)$$

where $\delta^{(\epsilon)}(x) = \begin{cases} \frac{1}{\epsilon}, & -\frac{\epsilon}{2} < x < \frac{\epsilon}{2} \\ 0, & |x| > \frac{\epsilon}{2} \end{cases}$



b) $\delta(x)$ can also be defined using following integral equations.

$$\int_{-\infty}^{+\infty} f(x) \delta(x) dx = f(0)$$

$$\int_{-\infty}^{+\infty} f(x) \delta(x-a) dx = f(a)$$

→ δ -function is not a function in usual mathematical sense.

→ It can be expressed as the limit of analytical

functions such as

$$\delta(x) = \lim_{\epsilon \rightarrow 0} \frac{\sin(\frac{x}{\epsilon})}{\pi x}$$

$$\delta(x) = \lim_{a \rightarrow \infty} \frac{\sin^2(ax)}{\pi ax^2}$$

$$\delta(x) = \lim_{\epsilon \rightarrow 0} \frac{1}{\pi} \frac{\epsilon}{x^2 + \epsilon^2} \text{ , etc}$$

~~Important properties:~~

Summary!

$$\delta(x) = \begin{cases} 0 & \text{if } x \neq 0 \\ \infty & \text{if } x = 0 \end{cases}$$

$$\text{and } \int_{-\infty}^{+\infty} \delta(x) dx = 1.$$

Important properties:

- (i) δ -function is an even function: $\delta(+x) = \delta(x)$
or $\delta(x-a) = \delta(a-x)$

$$(ii) \int_a^b f(x) \delta(x-x_0) dx = \begin{cases} f(x_0) & \text{if } a < x_0 < b \\ 0 & \text{elsewhere.} \end{cases}$$

(iii) $\delta(x) = 0$, for $x \neq 0$!

(iv) $x \delta(x) = 0$.

(v) $\delta(ax) = \frac{1}{|a|} \delta(x)$, for $a \neq 0$

(vi) $f(x) \delta(x-a) = f(a) \delta(x-a)$

(vii) $\int_c^d \delta(a-x)\delta(x-b)dx = \delta(a-b)$ for $c \leq a \leq d, c \leq b \leq d$

(viii) $\int_a^b \delta(x)dx = 1$ for $a \leq 0 \leq b$

(ix) $\delta[g(x)] = \sum_i \frac{1}{|g'(x_0)|} \delta(x-x_0)$

$x_0 \rightarrow$ zero of $g(x)$ and $g'(x_0) \neq 0$.

e.g. (a) $\delta(x^2-a^2)$

here $g(x) = x^2 - a^2$

$x^2 - a^2 = 0$

$\Rightarrow x = \pm a$

$g'(x) = 2x$

$g'(x)|_{x=+a} = 2a$

$g'(x)|_{x=-a} = -2a$

$\therefore \delta(x^2-a^2) = \sum_i \frac{1}{|g'(x_0)|} \delta(x-x_0)$

$= \sum_{x_0=+a} \frac{1}{|g'(x_0)|} \delta(x-x_0) + \sum_{x_0=-a} \frac{1}{|g'(x_0)|} \delta(x-x_0)$

$= \frac{1}{|2a|} \delta(x-a) + \frac{1}{|2a|} \delta(x+a)$

$\delta(x^2-a^2) = \frac{1}{2|a|} [\delta(x-a) + \delta(x+a)]$

H.W. Obtain expression for $\delta[(x-a)(x-b)]$ following the above example.